B.Sc. Semester-V Examination, 2022-23

MATHEMATICS [Honours]

Course ID : 52112 **Course Code : SH/MTH/502/C-12**

Course Title : Group Theory-II

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer any **five** questions: 2×5=10

a) If ϕ be an automorphism of a group G, then prove that $H=\{x\in G:\phi(x)=x\}$ is a subgroup of G.

b) Show that if a group G is abelian, then $Inn(G)=\{e\}$.

c) Show that $\mathbb{Z}_2\times\mathbb{Z}_3$ is isomorphic to $\mathbb{Z}_6$.

d) Find all the abelian groups of order 1125.

e) Find the class equation of the group D_3.

f) Find the number of subgroups of order 25 in a group of order 75.

[Turn Over]

g) Let G be a group that has only two conjugate classes. Then prove that the order of G is 2.

h) Let H and K be two subgroups of a group G. Then prove that $N_k(H)$ is a subgroup of K, where $N_k(H)$ is the normalizer of H in K.

2. Answer any **four** questions: 5×4=20

a) Define characteristic subgroup and prove that the centre of a group is characterstic. 1+4

b) Show that the mapping $\phi(a+ib)=a-bi$ is an automorphism of the group of complex numbers under addition.

c) Let H and K be two subgroups of a group G such that $G = H \times K$. Then prove that G/K is isomorphic to H and G/H is isomorphic to K.

d) Can the cyclic group Z_{12} be expressed as the internal direct product of two proper subgroups? Justify.

e) i) If H and K are conjugate, then prove that $H\cong K$.

ii) If a group G has only one p-Sylow subgroup H, then show that H is normal in G.

3+2

f) i) If G is a group of order 48, then show that the intersection of any two distinct Sylow 2-subgroup of G has order 8.

ii) How many Sylow 5-subgroups of S_5 are there? Justify your answer. 3+2

3. Answer any **one** question: 10×1=10

a) i) If G is infinite cyclic group, then find $Aut(G)$.

ii) Prove Cauchy's theorem for a non-abelian group.

iii) If G is a group and G' be its commutator subgroup of G, then prove that the subgroup G' is normal in G. 4+4+2

b) i) Prove that $|Aut\ \mathbb{Z}_2 \times \mathbb{Z}_2| = 6$.

ii) How many elements of order 9 does $\mathbb{Z}_3 \times \mathbb{Z}_9$ have?

iii) Using Sylow's theorems, show that any group of order 175 is simple. 4+3+3

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